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Bartłomiej Błaszczyszyn

On scattering moments of marked point processes

Joint with A. Brochard, S. Mallat and S. Zhang

Scattering moments [1] is a discrete family of nonlinear and noncommuting operators computing at different scales the modulus of a wavelet transform of a one- or higher-dimensional signal (e.g. image). They are invariant with respect to translations and Lipschitz-continuous with respect to smooth diffeomorphisms of the signal. These properties make them useful in signal processing, in particular in relation to statistical learning.

First, we show how the scattering moments can be used to address the following problem of statistical learning of geometric marks of point processes, studied in [2]: An unknown marking (score) function, depending on the geometry of point pattern, produces characteristics (marks) of the points. One aims at learning this function from the examples of marked point patterns in order to predict the marks of new point patterns.

Next, we present some limit results for the scattering moments of marked point processes as the scale of the wavelets becomes small or big. The latter involves the central limit theorem for geometric statistics of point processes [3] and can be used to estimate the variance asymptotic of the underlying processes.

References

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[2] A. Brochard, B. Błaszczyszyn, S. Mallat, and S. Zhang (2019) Statistical learning of geometric characteristics of wireless networks. *Proc. of IEEE INFOCOM*; *arXiv:1812.08265*

[3] B. Błaszczyszyn, D. Yogeshwaran, and J. Yukich (2019) Limit theory for geometric statistics of point processes having fast decay of correlations, *Ann. Probab.* **47**, pp. 835–895; *arXiv:1606.03988*

Rémi Bardenet

DPPs everywhere: repulsive point processes for Monte Carlo integration, signal processing and machine learning

Joint with Ayoub Belhadji, Pierre Chainais, Julien Flamant, Guillaume Gautier, Adrien Hardy and Michal Valko

Determinantal point processes (DPPs) are specific repulsive point processes, which were introduced in the 1970s by Macchi to model fermion beams in quantum optics. More recently, they have been studied as models and sampling tools by statisticians and machine learners. Important statistical quantities associated to DPPs have geometric and algebraic interpretations, which makes them a fun object to study and a powerful algorithmic building block.

After a quick introduction to determinantal point processes, I will discuss some of our recent statistical applications of DPPs. First, we used DPPs to sample nodes in numerical integration, resulting in Monte Carlo integration with fast convergence with respect to the number of integrand evaluations. Second, we used DPP machinery to characterize the distribution of the zeros of time-frequency transforms of white noise, a recent challenge in signal processing. Third, we turned DPPs into low-error variable selection procedures in linear regression.

Viktor Beneš

Limit theorems for Gibbs particle processes including facet processes

Joint with Christoph Hofer-Temmel, Günter Last and Jakub Večeřa

A stationary Gibbs particle process Ξ in the Euclidean space, with deterministically bounded particles, defined in terms of higher-order potentials and an activity parameter, is studied. For small activity parameters, we can prove the mean value and variance asymptotics and the central limit theorem for admissible U-statistics of Ξ. An application of theoretical results to facet processes is presented.

Chinmoy Bhattacharjee

Convergence to Scale Invariant Poisson Processes

Joint with Ilya Molchanov

We study weak convergence of a sequence of point processes to a simple point process with particular emphasis on scale invariant point processes as limits. For two deterministic sequences $(z_n)_{n\geq 1}$ and $(s_n)_{n\geq 1}$ increasing to infinity as $n \to \infty$ and a sequence $(X_k)_{k\geq 1}$ of independent integer-valued random variables, we consider the sequence of random measures

$$\nu_n = \sum_{k=1}^{\infty} X_k \delta_{z_k/s_n}$$

and prove that under certain general conditions, it converges vaguely in distribution to the scale invariant Poisson process η on $\mathbb{R}_+$ with intensity $1/x$. An important motivating example relies on choosing $X_k \sim \mathrm{Geom}(1-1/p_k)$ and $z_k = s_k = \log p_k$ where $(p_k)_{k\geq 1}$ is an enumeration of the primes in increasing order. We derive a general result on convergence of the integrals $\int_0^1 x d\nu_n$ to the integral $\int_0^1 x d\eta$, thus providing a new way of proving certain Dickman convergence results available in the literature.

We then extend our results to multivariate settings and provide sufficient conditions for vague convergence in distribution for a certain broad class of sequences of random vectors that uplift points from the line to the space. This relies on an extension of the convergence results on the line to integrals of functions with unbounded supports.

Christophe A.N. Biscio

A general central limit theorem and subsampling variance estimator for α-mixing multivariate point processes.

Joint with Rasmus Plenge Waagepetersen

Central limit theorems for multivariate summary statistics of α-mixing spatial point processes have usually been established using either the so-called Bernstein's blocking technique or an approach based on [3]. It is characteristic that essentially the same theorems have been (re)-invented again and again for different specific settings and statistic considered. Moreover, although there exists estimates in some particular cases, the asymptotic variance is usually unknown or difficult to compute.

In this talk, we present a unified framework based on [3] to state, once and for all, a general central limit theorem for α-mixing multivariate point process that applies in a general non-stationary setting and is also applicable to non-parametric kernel estimators depending on a bandwidth converging to zero. In particular, we argue why this approach is more suitable that the one using Bernstein's blocking technique. We believe this can save a lot of work and tedious repetitions in future applications of α-mixing point processes.

Finally, we present a subsampling estimator of the asymptotic variance in central limit theorems. Our estimator is very flexible and model free. We illustrate its use in connection to confidence interval of estimators obtained by composite likelihood method for several non stationary point processes that may be regular or clustered.

Our results are available in [1] and [2].

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Antoine Brochard

Point process generative model with wavelet phase harmonics

Joint with Bartłomiej Błaszczyszyn, Stéphane Mallat and Sixin Zhang

This work aims at presenting a method for tackling the following problem: given the finite realization ϕ of a planar point process $\Phi \in \mathbb{M}(\mathbb{R}^2)$ with unknown distribution, we want to generate (approximations of) realizations of Φ. Based on the assumptions that the sample ϕ is large enough and that Φ is ergodic, our approach consists in building a family of descriptors $\{S_k(\phi),\ k \in \Lambda\}$, such that $S_k(\phi) \simeq \mathbb{E}(S_k(\Phi))$. If the descriptors are informative enough, they characterize the distribution, and any $\mu \in \mathbb{M}(\mathbb{R}^2)$ satisfying $S_k(\mu) \simeq \mathbb{E}(S_k(\Phi))$ is a good approximation of a realization of Φ. We then generate an approximate sample of Φ by first generating a sample $\nu = \sum_i \delta_{x_i}$ from an homogeneous Poisson point process, and use a gradient descent algorithm to find the positions $\{x_i^*\}$ that minimize $\sum_k |S_k(\phi) - S_k(\sum_i \delta_{x_i^*})|^2$. The descriptors we use in this work are based on wavelet transform, and are called wavelet phase harmonics [1]. They estimate the correlation of wavelet transforms across scales, with frequency transpositions. These descriptors are related to the correlations of feature maps of non-linear rectifier layers in deep neural networks. We study their ability to capture the distribution of highly structured (Poisson cluster, Cox) point processes, as well as the pertinence of the gradient descent method in generating approximations of point processes of a given distribution, by computing several statistics for the generated samples. The moduli of the wavelet transforms, called scattering moments, have already been used in [2] to statistical learning of geometric marks of point processes.

References

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Pierre Calka

Convex hulls of perturbed random point sets

Joint with J. E. Yukich

We start from a random point set which is in convex position and we add independently a random perturbation to each point, either uniform or Gaussian. This perturbation may be considered as an error or a noise. We then investigate the convex hull of this new random input and in particular its number of k-dimensional faces. The growth rate depends on the 'size' of the perturbation measured by a certain power and we identify the exact phase transitions. In each regime, we show scaling limits, expectation and variance asymptotics as well as a central limit theorem.

Valentina Cammarota

Boundary effect on the nodal length for Arithmetic Random Waves, and spectral semi-correlations

Joint with Oleksiy Klurman and Igor Wigman

We test M. Berry's ansatz on nodal deficiency in presence of boundary. The square billiard is studied, where the high spectral degeneracies allow for the introduction of a

Gaussian ensemble of random Laplace eigenfunctions ("boundary-adapted arithmetic random waves"). As a result of a precise asymptotic analysis, two terms in the asymptotic expansion of the expected nodal length are derived, in the high energy limit along a generic sequence of energy levels. It is found that the precise nodal deficiency or surplus of the nodal length depends on arithmetic properties of the energy levels, in an explicit way.

To obtain the said results we apply the Kac-Rice method for computing the expected nodal length of a Gaussian random field. Such an application uncovers major obstacles, e.g. the occurrence of "bad" subdomains, that, one hopes, contribute insignificantly to the nodal length. Fortunately, we were able to reduce this contribution to a number theoretic question of counting the "spectral semi-correlations", a concept joining the likes of "spectral correlations" and "spectral quasi-correlations" in having impact on the nodal length for arithmetic dynamical systems.

This work rests on several breakthrough techniques of J. Bourgain, whose interest in the subject helped shaping it to high extent, and whose fundamental work on spectral correlations, joint with E. Bombieri, has had a crucial impact on the field.

Elisabetta Candellero

Coexistence of First passage percolation processes on hyperbolic graphs

Joint with Alexandre Stauffer

We consider two first-passage percolation processes FPP_1 and FPP_λ, spreading with rates 1 and $\lambda > 0$ respectively, on a non-amenable hyperbolic graph G with bounded degree. FPP_1 starts from a single source at the origin of G, while the initial con figuration of FPP_λ consists of countably many seeds distributed according to a product of iid Bernoulli random variables of parameter $\mu > 0$ on $V(G) \setminus \{o\}$. Seeds start spreading FPP_λ after they are reached by either FPP_1 or FPP_λ. We show that for any such graph G, and any fixed value of $\lambda > 0$ there is a value $\mu_0 = \mu_0(G, \lambda) > 0$ such that for all $0 < \mu < \mu_0$ the two processes coexist with positive probability. This shows a fundamental difference with the behavior of such processes on $\mathbb{Z}^d$.

Jean-François Coeurjolly

Second-order variational equations for spatial point processes with a view to pair correlation function estimation

Joint with Francisco Cuevas-Pacheco and Rasmus Waagepetersen

Moments of counts of objects for spatial point processes are typically expressed in terms of so-called joint intensity functions or Papangelou conditional intensity functions which are defined via the Campbell or Georgii-Nguyen-Zessin equations. In this talk, I will consider a third type of equation called variational equations introduced for parameter estimation in Markov random fields by [1]. A key feature of variational equations which have been proposed for spatial point processes compared to Campbell and Georgii-Nguyen-Zessin equations is that they are formulated in terms of the gradient of the log intensity or conditional intensity function rather than the (conditional) intensity itself.

Our first contribution is to establish second-order variational equations for second-order reweighted stationary (or istotropic) spatial point processes. Since the new variational equations are based on the gradient of the log pair correlation function, they take a particularly simple form for pair correlation functions of log-linear form. Our second contribution is to use this new equation to propose a new non-parametric estimator of the pair correlation. Standard estimators are the kernel density estimator or the recent orthogonal series estimator proposed by [2]. One of the drawback of these estimators is that they cannot be guaranteed to be non-negative. We propose to use our second-order variational equation to estimate coefficients in an orthogonal series expansion of the log pair correlation function. This ensures that the resulting pair correlation function estimator is non-negative. I will show on simulations and on real data how this new estimator compares to existing ones.

References

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Peter F. Craigmile

Enhancing statistical inference for stochastic processes using modern statistical methods

Joint with Radu Herbei, Huong Nguyen, Matthew Pratola, Grant Schneider and C. Devon Lin

Stochastic processes such as stochastic differential equations (SDEs) and Gaussian processes are used as statistical models in many disciplines. However there are many situations in which a statistical design or inference problem associated with these processes is intractable, and approximations are then required. Traditionally these approximations often come without measures of quality.

We motivate using three examples:

(i) Approximating intractable likelihoods for SDEs;

(ii) Using "near-optimal design" to find spatial designs that minimize integrated mean square error;

(iii) Using well-designed data subsets to enhance stochastic gradient descent (SGD) for big data statistical learning.

We demonstrate approaches to framing such problems from a statistical perspective so that we can probabilistically quantify uncertainties when making approximations. Depending on the problem, we achieve this using a range of modern statistical methods such as Gaussian processes, point processes, sampling theory, sequential design, and quantile regression.

Olof Elias

The fractal cylinder model

Joint with Erik Broman. Filipe Mussini and Johan Tykesson

We consider a statistically semi-scale invariant collection of bi-infinite cylinders in $\mathbb{R}^d$, chosen according to a Poisson line process of intensity λ. The complement of the union of these cylinders is a random fractal which we denote by $\mathcal{V}$. This fractal exhibits long-range dependence, complicating its analysis.

Nevertheless, we show that this random fractal undergoes two different phase transitions. First and foremost we determine the critical value of λ for which $\mathcal{V}$ is non-empty.

We additionally show that for $d \geq 4$ this random fractal exhibits a connectivity phase transition in the sense that the random fractal is not totally disconnected for lambda small enough but positive.

For $d = 3$ we obtain a partial result showing that the fractal restricted to a fixed plane is always totally disconnected.

An important tool in understanding the connectivity phase transition is the study of a continuum percolation model which we call the fractal random ellipsoid model. This model is obtained as the intersection between the semi-scale invariant Poisson cylinder model and a k-dimensional linear subspace of $\mathbb{R}^d$. Moreover, this model can be understood as a Poisson point process in its own right with intensity measure $\ell_k \times \mu_E$ where ℓ_k denotes the Lebesgue measure on $\mathbb{R}^k$ and μ_E is the shape measure describing the random ellipsoid.

Mohammad Ghorbani

Functional marked point processes – A natural structure to unify spatio-temporal frameworks and to analyse dependent functional data

Joint with O. Cronie, J. Mateu, and J. Yu

This paper treats functional marked point processes (FMPPs), which are defined as marked point processes where the marks are random elements in some (Polish) function space. Such marks may represent e.g. spatial paths or functions of time. To be able to consider e.g. multivariate FMPPs, we also attach an additional, Euclidean, mark to each point. We indicate how the FMPP framework quite naturally connects the point process framework with both the functional data analysis framework and the geostatistical framework; in particular, we define spatio-temporal geostatistical marking for point processes. We further show that various existing stochastic models fit well into the FMPP framework, in particular marked point processes with real valued marks. To be able to carry out non-parametric statistical analyses for functional marked point patterns, we study characteristics such as product densities and Palm distributions, which are the building blocks for summary statistics. We proceed to define a new family of summary statistics, so-called weighted marked reduced moment measures, in order to study features of the functional marks. We derive non-parametric estimators for these summary statistics and, in addition, we show how other existing (marked and/or inhomogeneous) summary statistics may be obtained as special cases of these summary statistics. We finally apply these statistical tools to analyse the population structure such as demographic evolution and sex ratio over time in Spain provinces.

Ana I. Gomez

Modeling the covariogram in 2D length estimation

The variance under systematic sampling on the circle has been widely studied by L.M. Cruz-Orive and X. Gual-Arnau. Several formulas for different methods in local stereology are based on a given model for the covariogram. Quasi Monte Carlo methods and the Moore-Aronszajn theorem provide alternative models to attack this problem.

In this work, I focus on the special case of the variance predictor for the estimator of planar curve length based on intersection counting with a square grid (Buffon -Steinhaus estimator), and propose improvements to increase the accuracy of the predictions. Finally, the approach is tested against Monte Carlo replications in an enlarged dataset of curvilinear features used in the literature and the results and further applications are discussed.

Lothar Heinrich

Some older and newer results on Brillinger-mixing point processes

We consider a stationary infinite-order point process $\Psi \sim P$ on $\mathbb{R}^d$ satisfying the additional assumption that, for each $k \geq 2$, the reduced kth-order factorial cumulant measure $\gamma_{red}^{(k)}(\cdot)$ has finite total variation $||\gamma_{red}^{(k)}||_{TV}$ on $(\mathbb{R}^d)^{k-1}$. This property of Ψ, which is attributed to D. R. Brillinger, expresses weak mutual correlations between the numbers of atoms taking the counting measure Ψ in distant sets. This condition allows to prove asymptotic normality of shot noise processes, higher-order moment measure estimators (e.g. the empirical K—function), empirical product densities etc. Furthermore, CLTs for hyperplane processes driven by a one-dimensional B-mixing pp can be established. $\Psi \sim P$ is called *strongly B-mixing* if $||\gamma_{red}^{(k)}||_{TV} \leq a^k\, k!$ for some $a > 0$ and any $k \geq 2$. We show that a *B-mixing* pp $\Psi \sim P$ is *mixing* iff P is uniquely determined by its one-dimensional moment sequences. If there is no uniqueness then $\Psi \sim P$ need not be *ergodic* in general, see [1]. On the other hand, if the pp $\Psi \sim P$ is strongly B-mixing then its tail-σ-algebra of Ψ is trivial. Finally, we show that a Log-Gaussian Cox process is B-mixing if the covariance function $c(\cdot)$ of the underlying stationary Gaussian field is absolutely integrable and conditions are formulated implying that a renewal pp is strongly B-mixing.

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Felix Herold

Tessellations in hyperbolic space

Joint with Collaborators

Random tessellations in d-dimensional Euclidian space have been studied intensively in the last few decades. Famous models are for example hyperplane tessellations, Voronoi tessellations or STIT tessellations. Each of these models (and their combinations) has a variety of applications, for instance in telecommunication, biology or geography. Recent work aims to transferring known results into spherical space for further applications and a deeper understanding of the connection between probability and geometry. To follow this line of research this talk will deal with the construction of random tessellations in d-dimensional hyperbolic space. Further on we will give some first results in the hyperbolic setting of classical problems considered in stochastic geometry.

Christian Hirsch

Optimal stationary markings

Joint with Bartek Błaszczyszyn

How to describe hard-core thinnings of stationary particle processes, which maximize the volume fraction? How to choose the transmission powers in a network of base stations so as to maximize the global throughput? What is the optimal placement of content in a network of spatially distributed caches? All of these questions can be cast into the framework of optimal stationary markings of point processes.

We offer intensity-optimal and locally optimal markings as two possible formalizations of this concept. We discuss the question of existence and under which conditions the two notions coincide. By forming connections to stabilization techniques from stochastic geometry, many of the core ideas from the special setting of maximal hard-core thinnings [1] can be extended to optimal markings. Moreover, the more general setting also gives rise to novel examples of uniqueness and non-uniqueness.

References

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Jürgen Kampf

The variances of surface area estimators based on pixel configuration counts

In this talk we deal with the estimation of the surface area of sets of which only a pixel image is observed. So-called *local algorithms* are quite popular, since they are computationally fast and easy to implement. These algorithms count the number of occurrences of small patters, called *pixel configurations*, in the image and estimate the surface area of the underlying set by a linear combination of these counts.

The asymptotic relative biases of local algorithms for the surface area in $\mathbb{R}^3$ have been investigated by Ziegel and Kiderlen, when the underlying set is shifted at random and the lattice distance tends to zero.

But the bias is only one component of the error. The other component – the variance – has not been considered so far. In this talk we will show that under the setup described above the variance tends to zero of quadratic order and thus is neglectable compared to the bias.

In order to prove this result we derive deterministic upper and lower bounds for the number of occurrences of the pixel configurations in pixel images of sufficiently regular sets. These bounds are close to each other, which implies that the variances of these counts are small and thus that the variance of the surface area estimator is small.

Kateřina Koňasová

Classification task in the context of replicated point patterns

Joint with Jiří Dvořák

Classification task, one of the fundamental tasks in machine learning and also in statistics, aims to classify a new observation to the one of the k possible classes. In the point pattern setting, its purpose is to label the incoming observed pattern with one of the k possible labels.

For solving the classification task numerous methods are available. Some of them are related with measuring dissimilarities between investigated objects. In the point pattern setting, there exist various metrics that can be used as a dissimilarity measure (e.g. the Hausdorff metric), but most of them ignore some important properties of the underlying point pattern data.

We propose a non-parametric approach to solving the classification task in the context of replicated point patterns using the kernel regression method. In this case we consider a semimetric based on functional summary characteristics as a dissimilarity measure. The employment of functional summary characteristics could be quite advantageous, since they contain a valuable information about the geometrical structure hidden in the investigated data. Performance of this method will be illustrated by means of simulation study and some examples of the classification task for real point pattern data will be discussed.

Günter Last

Exponential decorrelation of subcritical repulsive Gibbs particle processes

Joint with Viktor Beneš, Christoph Hofer-Temmel and Jakub Večeřa

We consider a stationary Gibbs particle process with deterministically bounded particles on Euclidean space defined in terms of a non-negative pair potential and an activity parameter. For small activities we show that the correlation functions factorize in an exponentially decreasing way. Our main technical tool is a disagreement coupling of two Gibbs processes with a dominating Poisson particle process. This coupling is based on a spatial (non-dynamical) thinning construction of finite Gibbs processes. We will provide a general setting for such a thinning which applies also to non-repulsive Gibbs processes. Our results can be used to establish uniqueness of Gibbs distributions as well as central limit theorems.

Frédéric Lavancier

Asymptotic inference of determinantal point processes

Determinantal point processes (DPPs) are stochastic processes involving negative dependencies. Defined on a finite discrete space, they have been used in machine learning and survey sampling design to generate subsets of objects exhibiting diversity. Defined on a continuous space (typically $\mathbb{R}^d$), they provide useful models for the simulation and the description of repulsive spatial point processes. The increasing popularity of DPPs is mainly due to the fact that they are flexible models, through the choice of their kernel, and that both their moments and their density on a compact set are explicitly known. We focus in this talk on the (increasing domain) asymptotic properties of continuous DPPs. As a matter of fact, DPPs turn out to be Brillinger mixing, α-mixing, β-mixing, and negatively associated. These nice mixing properties allow us to get general central limit theorems for functionals of a DPP. As an application, we derive the asymptotic properties of estimating function estimators of a (possibly non-stationary) parametric DPP. This setting includes contrast estimators based on the K-function or the pair correlation, Palm likelihood estimator and composite likelihood estimator. We also discuss likelihood inference, for which the asymptotic properties remain challenging to establish. Nevertheless, we prove under some conditions the consistency of the maximum likelihood estimator of a stationary DPP. We further introduce an approximation of the likelihood, that does not require the spectral decomposition of the kernel and proves to be asymptotically equivalent to the true likelihood. This talk is based on several joint works with Christophe Biscio, Bernard Delyon, Jesper Møller, Arnaud Poinas, Ege Rubak and Rasmus Waagepetersen.

Jesper Møller

The structure of stationary time series and point processes when constructing singular distribution functions

Joint with Horia Cornean, Ira W. Herbst, Benjamin Støttrup and Kasper S. Sørensen

A function $F : \mathbb{R} \to \mathbb{R}$ is singular if it is non-constant and $F'(x) = 0$ for Lebesgue almost all $x \in \mathbb{R}$ (sometimes further conditions are imposed). We construct rich classes of singular cumulative distribution functions (CDF) F for random variables X on the unit interval $[0, 1]$. Our basic idea is to construct such functions by considering a q-adic expansion of X, where $q \in \mathbb{N}$, and where the coefficients in the expansion form a time series as follows. Let $X_1, X_2, \ldots$ be a time series with values in $\{0, 1, \ldots, q-1\}$. Then define $X := \sum_{n=1}^{\infty} X_n q^{-n}$.

In particular, we completely characterize the CDF when $\{X_n\}_1^\infty$ is stationary; or equivalently when the point process $\sum_1^\infty \delta_{X_n}(\cdot)$ is stationary (here, δ_t is the Dirac measure at t). In fact F becomes a mixture of three CDFs F_1, F_2, F_3 on $[0, 1]$, where F_1 is the uniform CDF on [0,1]; F_2 is singular discrete and is a mixture of countable many CDFs, each of them being uniform on a finite set of so-called purely repeating q-adic numbers which are members of a cycle (we clarify how this corresponds to a stationary cyclic Markov chain of order equal to the length of the cycle); and F_3 is singular continuous.

Two simple models are well-known: Take $\{X_n\}_{n\geq 1}$ to be independent identically distributed. In the dyadic case $q = 2$, if 0 and 1 equally likely, then dF is just Lebesgue measure on $[0, 1]$. In the triadic case $q = 3$, if 0 and 2 are equally likely and $\mathrm{P}(X_1 = 1) = 0$, then F is the well known Cantor function.

These models and several others are discussed more fully in the talk. Moreover, we demonstrate in several examples that continuity of F is a natural property when considering specific models for stationary time series and point processes.

Kilian Matzke

Triangle Condition for the Critical Random Connection Model in High Dimensions via Lace Expansion

Joint with Markus Heydenreich, Remco van der Hofstad and Günter Last

The random connection model is a random graph whose vertices are given by the points of a Poisson process and whose edges are obtained by randomly connecting pairs of Poisson points in a position dependent but independent way. Under very general conditions, the resulting random graph undergoes a percolation phase transition if the the Poisson density varies, and we are interested in the case of critical percolation. Our main result is an infrared bound for the critical connectivity function if the dimension is sufficiently large or if the pair connection function has sufficient slow decay. This is achieved through an adaptation of the percolation lace expansion for Poisson processes.

Adrien Mazoyer

Monte-Carlo integration in any dimension using a single determinantal point process

Joint with Jean-François Coeurjolly and Pierre-Olivier Amblard

In a recent work [1], a particular projection determinantal point process (DPP), called Orthogonal Polynomial Ensemble (OPE), has been proposed to produce exactly n quadrature points to estimate $\int_{[0,1]^d} f_d(u)\mu(\mathrm{d}u)$ where, for some $d \geqslant 1$, f_d is a d-dimensional μ-measurable function. The authors proved that their estimator satisfies a central limit theorem with explicit variance and rate of convergence $\sqrt{n^{1+1/d}}$ instead of the typical $\sqrt{n}$, under the main assumption that f_d is continuously differentiable and compactly supported.

Using a product of Dirichlet type kernels instead of orthogonal polynomials, we extend this work when μ is the Lebesgue measure: to mimic a problem encountered in computer experiments, we investigate the problem of estimating $\mathscr{I}_\omega = \int_{[0,1]^\omega} f_\omega(u)\mathrm{d}u$ using a single configuration of d-dimensional points. We prove that our Dirichlet model is actually well-suited to such a problem, by exploiting the fact that any projection on a lower dimensional space of this model is distributed as a particular α-DPP. For any $\omega = 1,\dots,d$, we eventually exhibit estimator of $\mathscr{I}_\omega$ which satisfies a central limit theorem with explicit variance and rate of convergence $\sqrt{n^{1+1/d}}$. Moreover, we relax the required assumptions for the function f_ω: we only require that f_ω is “half”-differentiable (an assumption which is for instance satisfied for the L^1-norm) and do not impose that f_ω is compactly supported.

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Ilya Molchanov

Random diagonal transformations of convex bodies

Joint with Felix Nagel

The family of convex bodies obtained from a given centred convex body K by applying rotation and scaling and taking Minkowski sums is called the Minkowski class generated by K. In this talk we replace rotations with transformations of K by diagonal matrices u. The obtained family of convex bodies can be described as expectations of random sets ξK, where $\xi \in \mathbb{R}^d$ determines the diagonal elements of the matrix transforming K. If K is a segment with end-points $\pm(1,\dots,1)$, one obtains the family of zonoids.

It is shown that the diagonal Minkowski class is dense if and only if the equality of expectations $\mathbf{E}(\xi K) = \mathbf{E}(\eta K)$ for two integrable random vectors ξ and η implies the zonoid equivalence of ξ and η, that is, $\mathbf{E}|\langle \xi, u\rangle| = \mathbf{E}|\langle \eta, u\rangle|$ for all u. In this case, K is called D-universal. Conditions ensuring the D-universality of K are provided. It is shown that expectations of diagonally scaled ℓ_p-balls naturally arise in relation to totally skewed strictly stable laws in Euclidean space.

Franz Nestmann

Cluster counting in the random connection model

Joint with Günter Last and Matthias Schulte

The classical random connection model is a spatial random graph, whose vertices are given by a stationary Poisson point process η in $\mathbb{R}^d$. In order to generate the edges of this graph, a connection function $\varphi\colon \mathbb{R}^d \to [0,1]$ is required. Each pair of distinct vertices $x, y \in \eta$ is connected via an edge with probability $\varphi(x-y)$, independently of all other vertices and connections.

This model can be extended by adding a random and independent mark in $[0, \infty)$ to each vertex. In this marked model, the probability for drawing an edge between two Poisson points also depends on the marks of the two points. A notable example is the Gilbert graph with random radii. Here the marks of the vertices are interpreted as radii of spheres around the points. Two vertices are connected by an edge if and only if the corresponding spheres intersect.

We are interested in the number of components of the (marked) random connection model that are isomorphic to a given finite connected graph and are located in a compact convex observation window. In addition, we consider the total number of finite components within the observation window. For increasing observation windows asymptotic variances are studied and quantitative central limit theorems are shown. These results are based on bounds for the normal approximation of functionals of pairwise marked Poisson point processes that are derived from the Stein-Malliavin method.

Florian Pausinger

Persistent Betti numbers of random Čech complexes

Joint with Ulrich Bauer

We study the persistent homology of random Čech complexes. Generalizing a method of Penrose for studying random geometric graphs, we first describe an appropriate theoretical framework in which we can state and address our main questions. Then we define the kth *persistent* Betti number of a random Čech complex and determine its asymptotic order in the subcritical regime. This extends a result of Kahle on the asymptotic order of the ordinary kth Betti number of such complexes to the persistent setting.

Zbyněk Pawlas

Limit theorems for marked particle processes

By a marked particle process we understand a simple point process in $\mathcal{K}' \times \mathbb{M}$, where $\mathcal{K}'$ denotes the space of non-empty compact subsets of $\mathbb{R}^d$ and $\mathbb{M}$ denotes the mark space. For stationary marked particle processes the intensity describes the mean number of particles (grains) per unit volume and the grain-mark distribution describes the joint distribution of a typical particle and its corresponding mark. We consider the estimators of these first-order characteristics and investigate their asymptotic behaviour as the observation window is expanding. For several particular models we are mainly interested in weak or strong consistency, variance asymptotics and asymptotic normality.

Mathew Penrose

Limit behaviour of the coverage threshold

Let $X_1, X_2, \ldots$ be i.i.d. random uniform points in a bounded domain $A \subset \mathbb{R}^d$. Define the *coverage threshold* R_n to be the smallest r such that A is covered by the balls of radius r centred on $X_1, \ldots, X_n$. Clearly R_n is random, and nonincreasing in n. We discuss the limiting behaviour of R_n as $n \to \infty$, including:

- limting distribution when A is polygonal or polyhedral;
- strong laws of large numbers when A has a smooth boundary;
- limiting behaviour for the number of record times in the sequence (R_n), i.e. instances when $R_n < R_{n-1}$.

The analysis relies on classical results by Hall and by Janson, along with a careful treatment of boundary effects. For some of our results, we can relax the requirement that the underlying density on A be uniform.

Anders Rønn-Nielsen

Limits and extremal behaviour of Lévy-based models

Joint with Eva B. Vedel Jensen

A continuous, infinitely divisible d-dimensional random field given as an integral of a kernel function with respect to a Lévy basis is considered. Under mild regularity conditions we derive central limit theorems for the moment estimators of the mean and the variogram of the field.

Under the supplementary assumption that the Lévy basis has a convolution equivalent Lévy measure, we derive an expression for the asymptotic probability that the supremum of the field exceeds the level x as this tends to infinity. A main result is that the asymptotic probability is equivalent to the right tail of the underlying Lévy measure. Furthermore, the asymptotic behaviour of the probability that an excursion set contains rotations of a given geometrical objects of fixed size, e.g. a ball or a line, is studied. This probability is similarly asymptotically described by the right tail of the Lévy measure.

Jan Rataj

On integral geometric formulas for excursion sets of random fields

If Z is a continuous stationary (isotropic) real-valued random field on the Euclidean space then the excursion set $A_r := \{t : Z(t) \geq r\}$ is a stationary (isotropic) random closed set for any r. If Z has a.s. C^2 trajectories then, under mild nondegeneracy assumptions, A_r has C^2 boundary and its curvature densities can be determined. In the particular case of zero mean stationary Gaussian processes, the curvature densities are functions of the second order partial derivatives of the covariance function at the origin and translative (kinematic) integralgeometric formulas can be used to obtain the mean values of the total k-th curvature $\mathbb{E}C_k(A_u \cap W)$ of A_u intersected with a bounded window W. A basic reference is the book of Adler and Taylor (Springer, 2007). Our aim is to discuss possible extensions of the results by relaxing the smoothness assumptions, using recent kinematic formulas of integral geometry.

Thomas Richthammer

How rigid are crystals in 2D? The hard disk model and 2D Gibbsian point processes.

Joint with Michael Fiedler

2D particle systems are believed to show some sort of crystalline phase: If the temperature is sufficiently low (or the density sufficiently high) the particles arrange themselves into a regular pattern, which is characterized by long-range correlations. So far there is no rigorous proof of this phenomenon.
Particle systems in equilibrium are usually modelled by Gibbsian point processes. We will describe this model in general and the simple special case of the hard disk model. We will give a short overview of what is known and what is expected to be true for these models. We will then describe recent results showing that the expected regular pattern within these models can not be too rigid: In a system of size n, positions of points near the center of the system fluctuate by at least a constant times $\sqrt{\log n}$. Our result holds for fairly general interaction potentials (including all interesting examples of interacting particle systems we know of) and arbitrary values of temperature and particle density.

Patrick Rubin-Delanchy

Spectral embedding of graphs

In this talk I will present recent statistical results on spectral embedding, or the representation of a graph as a point cloud using the spectral decomposition of its adjacency matrix (and other related structures such as the normalised Laplacian). In particular I will introduce a model, called the generalised random dot product graph, under which the points can be interpreted as latent position estimates. This allows us to make statements of uncertainty about the points in ordinary statistical language, for example, using confidence intervals. Popular community-based network models such as the mixed membership and standard stochastic block models are analysed as special cases, and we will also touch on estimating other forms of community structure using topological data analysis. Results are illustrated with cyber-security applications.

Matthias Schulte

Limit theorems for heavy-tailed Boolean models

The Boolean model Z is obtained as union of all grains of a stationary Poisson process of compact convex sets in $\mathbb{R}^d$. For a geometric functional ψ such as volume or surface area and a compact convex set W one is interested in the behaviour of $\psi(Z \cap W)$. For increasing inradius of the observation window W it is known that $\psi(Z \cap W)$ converges, after rescaling, in distribution to a standard Gaussian random variable if the second moments of the intrinsic volumes of the typical grain are finite. The focus of this talk is on a class of heavy-tailed Boolean models where the latter condition is violated. For this situation distributional limit theorems with alpha-stable limiting distributions are derived.

Hauke Seidel

Random Sections of Regular Polytopes and Convex Cones

Joint with Zakhar Kabluchko and Dmitry Zaporozhets

Let P be an n-dimensional regular cross-polytope, simplex, or cube centred at the origin of $\mathbb{R}^n$. We consider convex cones of the form

$$C = \{\lambda x + \lambda e_{n+1} : \lambda \geq 0, x \in P\} \subset \mathbb{R}^{n+1},$$

where $e_1, \dots, e_{n+1}$ is the standard basis of $\mathbb{R}^{n+1}$.

We shall derive explicit probabilistic expressions for the inner and outer solid angles and the intrinsic volumes of these cones. As a corollary, we shall derive a formula for the inner and outer solid angles of a regular crosspolytope.

The goal of the talk is to explain, how these cones are valuable tools in determining the expected number of faces of the intersection of a random linear subspace and a regular cross-polytope, cube or simplex.

A R Soltani

An application of the Geometry of Random Vectors in Data Analysis

Joint with S. M. Aboukhamseen

In this article we present a protocol to generate a data center, using the under lying geometry of random variables and random vectors. The geometry of random vectors enables us to establish simulation procedures to generate quite a rich data center. The idea is to match observed samples of the data to the data center. This technique appears to be quite powerful tool for data analysis, parametric and nonparametric estimation. The technique also appears to be a nice contribution to the field of machine learning and the nearest neighbor estimation method. The protocol can be expanded to generate data centers for the realizations of large classes of stochastic processes.

Mads Stehr

Extremal properties over time of an infinitely divisible random field with convolution equivalent Lévy measure

Joint with Anders Rønn-Nielsen

We consider a $(d+1)$-dimensional infinitely divisible random field indexed by space and time, and we find the asymptotic representation of the tail of its supremum. More specifically, we let the field be given as an integral of a kernel function with respect to a Lévy basis with convolution equivalent Lévy measure, and we show that the tail of the supremum is asymptotically equivalent to the tail of the Lévy measure. Actually, if Ψ is some appropriate operator, we show the more general result that the tail of Ψ applied to the field is asymptotically equivalent to the tail of the Lévy measure.

We define the kernel function in such a way that the field only depends on noise in the past, that is, the kernel is càdlàg in the time-coordinate. As part of the proof of the tail-behaviour, we also show that the field has a version with sample paths which are càdlàg in the time-coordinate.

Anne Marie Svane

Testing goodness of fit for point processes via topological data analysis

Joint with Christophe A. N. Biscio, Nicolas Chenavier and Christian Hirsch

In this talk, we present a central limit theorem for the persistence diagram associated with a 2D point process when the observation window becomes increasingly large. In order to apply general convergence results, a bounded version of persistent Betti numbers is introduced. The persistence diagram is shown to converge to a Gaussian process under suitable conditions on the point process.

The results are used to derive asymptotically normal test statistics to asses goodness of fit for point patterns. The power of these tests is investigated on simulated point patterns and we apply the tests to a real data set.

Vincent Tassion

Sharpness of the phase transition for continuum percolation

Joint with H. Duminil-Copin and A. Raoufi

In this talk we consider some percolation processes in dimension $d \geq 2$ constructed from Poisson processes (e.g. Boolean percolation, Voronoi percolation). In the case of Boolean percolation, we begin with a Poisson process of intensity λ in $\mathbb{R}^d$, and around each point of the process we draw a ball of random radius. We are interested in the connectivity properties of the *occupied* set, i.e. the set of points covered by at least one ball. Such process undergoes a phase transition at a critical intensity λ_c: for $\lambda < \lambda_c$, all the connected component of the occupied set are bounded a.s., and for $\lambda > \lambda_c$, there exists an unbounded connected component of occupied points. Using a new method based on the theory of randomized algorithms, we prove that the phase transition is *sharp* for a large class of continuum percolation processes, in the sense that the connection probabilities in the regime $\lambda < \lambda_c$ decay very fast.

Benjamin Taylor

Inference Under Obfuscation of Stochastic Processes

Joint with Various Collaborators

Inference under obfuscation of stochastic processes is a common modelling challenge that is often ignored, or substantially simplified in practice (sometimes for good reason). In this talk I will discuss three projects I have worked on in which this phenomenon occurs and some thoughts about how to overcome the challenges we encountered, covering ideas in various stages of intellectual development.

The first project, joint with Hugh Sturrock, Ricardo Andrade-Pacheco and Adam Bennett (USCF) concerns the modelling of case counts of malaria at the health facility level in Zambia. Health facilities have unknown catchment areas which report irregularly and change over time. We treat the underlying data-generation process as a spatially continuous point process and seek to explicitly capture the obfuscation process through an additional model hierarchy, with inference following from a GPU-accelerated data-augmentation scheme.

The second project, joint with Ruy Ribeiro (Universidade de Lisboa), Ashwin Balagopal (Johns Hopkins) and Paula Moraga (Lancaster/Bath) concerns the modelling of the propagation of hepatitis C (HCV) infection in the liver. We have data from a grid of cells collected from patients with HCV and I will talk about our increasing understanding of the nature of these data and potential ways we can proceed with their analysis.

The third project, joint with Jorge Mateu and Jonatan Gonzalez-Monsalve (Universitat Jaume I), we consider the modelling of the locations and sizes of bubbles in a flotation chamber that have arisen as a result of a chemical metal extraction process. I will talk about some of the subtleties in analysing data from this experiment.

Steffen Winter

On distributional properties of geometric functionals of fractal percolation

Joint with Michael Klatt.

Fractal percolation is a family of random self-similar sets suggested by Mandelbrot in the seventies to model certain aspects of turbulence. It exhibits a dramatic topological phase transition, changing abruptly from a dust-like set of isolated points to a system spanning cluster. The transition points are unknown and difficult to estimate, and beyond the fractal dimension not so much is known about its geometry.

We introduce geometric functionals for the fractal percolation process F. These random variables arise as suitably rescaled almost sure limits of intrinsic volumes of finite approximations of F. We establish the existence of these limit functionals and obtain in some cases explicit formulas for their expectations and variances as well as for their finite approximations. The approach is similar to fractal curvatures but in contrast the new functionals can be determined explicitly and approximated well from simulations.

D. Yogeshwaran

CLT for point processes with fast decay of correlations.

Joint with B. Blaszczyszyn, J. E. Yukich, S. Vadlamani and A. Tulasi Ram Reddy.

We shall try to understand some central limit theorems for quasi-local statistics of point processes with fast decay of correlations. In the talk, we shall consider the underlying space for the point process to be either Euclidean or Cayley graphs but shall try to hint at extensions to more general spaces. Fast decay of correlations is weaker than many mixing conditions for point processes and is satisfied by many interesting point processes. Quasi-locality will be quantified by what is known as a 'stabilizing radius'. We will see that under suitable tail assumptions on the 'stabilizing radius', fast decay of correlations of the point process, growth condition on balls in the underlying space and appropriate variance lower bounds, one can prove a CLT for quasi-local statistics of point processes. The proof technique via controlling mixed moments and thereby bounding the cumulants also allows for surface-order variance growth. Some point processes have surface-order variance growth and hence this flexibility with the variance growth is important. Some applications to statistics of spatial unimodular random graph models will be mentioned as well.

Joseph E. Yukich

Multivariate normal approximation for statistics in geometric probability

Joint with Matthias Schulte

We employ stabilization methods in the context of Malliavin-Stein theory to establish rates of multivariate normal convergence for a large class of vectors

$$(H_s^{(1)}, \ldots, H_s^{(m)}), \qquad s \geq 1,$$

of marked Poisson point processes in Euclidean space, as the intensity parameter $s \to \infty$. The rates are in terms of the d_2 and d_3 distances, a generalized multivariate Kolmogorov distance, as well as in terms of the convex distance defined in terms of indicators of convex sets. In general the rates are unimprovable. We use the general results to deduce presumably optimal rates of multivariate normal convergence for statistics arising in random graphs and topological data analysis as well as for multivariate statistics used to test equality of distributions.

Posters

Felix Ballani

Exact simulation of Poisson and Matérn particle processes

We first address exact simulation of stationary Poisson particle processes in the sense of avoiding edge effects, which possibly arise due to a bounded simulation window but not necessarily uniformly bounded particles. For almost surely convex particles this requires sampling from the V_k-weighted grain distributions for all intrinsic volumes $V_0, \dots, V_d$ [1]. Extending previous work, we give a rigorous result on how the V_k-weighted typical cell of a stationary Poisson hyperplane mosaic can be simulated.

Besides the obvious consequence that the exact simulation of Poisson particle processes is also helpful for that of related models like Boolean random functions or dead leaves models, it is even crucial for the exact simulation of derived Matérn particle processes [2]. Namely, since a Matérn particle process results from deleting some of the particles from a Poisson particle process in order to achieve a non-overlapping configuration, the decision on whether a primary particle survives requires knowledge of all primary particles hitting it. We discuss this point to some extent also for Matérn III type particle processes including further ideas from [3].

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Rikke Eriksen

Uniqueness of the measurement function in Crofton's formula

Joint with Markus Kiderlen

Crofton's intersection formula states that the $(n-j)$'th intrinsic volume of a compact, convex set in $\mathbb{R}^n$ can be obtained as an invariant integral of the $(k-j)$'th intrinsic volume of sections with k-planes. This poster will discuss the question if the $(k-j)$'th intrinsic volume can be replaced by other functionals, that is, if the measurement function in Crofton's formula is unique.

The answer is negative: we show that the sums of the $(k-j)$th intrinsic volume and certain translation invariant, continuous, valuations of homogeneity degree k yield counterexamples. If the measurement function is local, i.e. translation invariant, locally determined functionals [1], these turns out to be the only examples when $k=1$ or when $k=2$ and we restrict considerations to even measurement functions. Additional examples of local functionals will be constructed when $k \geq 2$.

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Daniela Flimmel

Central limit theorem for an unbiased estimator of cell characteristics of a weighted Voronoi tessellation

Joint with Joseph Yukich and Zbyněk Pawlas

Take $\mathscr{X}$ to be a random tessellation of any type, i.e. a random family of closed compact subsets of $\mathbb{R}^d$ (*cells*) with disjoint interiors, covering the whole space. We observe its realization in a bounded window $W \subset \mathbb{R}^d$. Based on this observation, we study its geometrical properties via functionals of the form of a sum of local contributions of each individual cell

$$H(\mathscr{X}) = \sum_{K \in \mathscr{X}} \xi(K, \mathscr{X}) \mathbf{1}\{K \subset W\},$$

where ξ are different test functions called *scores* (chosen so that the sum makes sense). This statistic, although, may disregard the edge effects caused by observing a realization in a bounded window, which can lead to a bias. One possibility how to treat the edge effects is to incorporate the minus sampling technique, i.e. we multiply each summand by a weight $\frac{|W|}{|W \ominus K|}$, where $W \ominus K = \{x \in \mathbb{R}^d : K + x \subset W\}$ and $|A|$ stands for the Lebesgue measure of A. It results in so called *Horvitz–Thompson estimator*, which is an unbiased estimator of the expected value of the functional ξ applied on the typical cell.

Many asymptotic results were shown for the this type of statistics for Poisson–Voronoi tessellation. We assume that the underlying Poisson process is independently marked with marks in $\mathbb{R}_+$, i.e. we assign each point with a positive weight. It allows us to study more flexible models of random tessellations (e.g. Laguerre and Johnson-Mehl tessellations). Letting the observation window tend to $\mathbb{R}^d$, we show a central limit theorem as well as the variance asymptotic for estimators of cell characteristics in the weighted Voronoi setting using the stabilization method.

Henning Höllwarth

On Confidence Regions for Parameters of Gibbs Point Process Models Based on the Maximum Likelihood Estimator

Maximum likelihood (ML) inference for parametric Gibbs point process models is highly desirable, however, it suffers from an intractable likelihood. Although an ML-estimate can be computed at least approximately, it requires intensive and sophisticated computations, especially for point processes with a strong dependence structure. Furthermore, just very little is known about the theoretical behavior of the MLE, besides its strong consistency (see [3]). Due to these obstacles, a corresponding confidence region were out of reach so far. On our poster we provide a fast method to construct a parametric bootstrap confidence region based on the MLE. To this end, we use simpler estimation methods like the maximum pseudolikelihood (see e. g. [2]) or the variational estimation procedure (see [1]). The discussion covers analytical and numerical justifications of our construction.

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Kateřina Helisová

Similarity of realisations of random sets via approximation by unions of convex compact sets

Joint with Vesna Gotovac and Ivo Ugrina

The poster concerns a similarity measure of realisations of random sets through a heuristic based on approximation by unions of convex compact sets, evaluation of the support functions of the approximating sets and consequent usage of envelope tests and N-distances. The measure is used to distinguish between two realisations, more precisely to decide whether two given realisations come from the same underlying process when we have their pixel images. The suggested procedure is justified through simulation studies of common random models like Boolean model and Quermass-interaction processes with different parameters.

Louis Gammelgaard Jensen

Cluster Marked Cluster Point Processes with applications in super resolution microscopy

Joint with Ute Hahn

Cluster Marked Cluster Point Processes (CMCpps) are a class of clustered, marked point process models with a simple dependence structure between locations and marks. They allow for modelling of clustered data where each cluster may depend on information provided by the mark of a (latent) parent process. CMCpps hold potential for modelling of data from super resolution microscopy, where photo-blinking artifacts give rise to clusters of multiple appearances from the same molecule. These blinking artifacts complicate matters of counting proteins, and quantifying the degree of clustering between them. On my poster, I will present a set of new methods that hold promise for direct estimation of blinking characteristics, such as the mean number of blinks per protein, the expected waiting time between blinks, and the cluster shape. The methods allow for splitting up the contribution from proteins and blinking artifacts, making possible the quantification of the degree of clustering or repulsiveness between individual proteins.

Sanjoy Kumar Jhawar

Percolation in some planar random graph models.

Joint with Srikanth K. Iyer

We study phase transition and percolation at criticality for three planar random graph models, viz., the homogeneous and inhomogeneous enhanced random connection model (eRCM and IeRCM) and the Poisson stick model. The model eRCM and IeRCM are extension of the random connection model (RCM). These models are built on a homogeneous Poisson point process $\mathscr{P}_\lambda$ in $\mathbb{R}^2$ of intensity λ. In the RCM, the vertices at $x, y \in \mathscr{P}_\lambda$ are connected with probability $g(|x-y|)$ independent of everything else, where $g : [0,\infty) \to [0,1]$ and $|\cdot|$ is the Euclidean norm. In the inhomogeneous version (IRCM) of RCM, the vertices in $\mathscr{P}_\lambda$ are endowed with random weights W, where $P(W > w) = w^{-\beta} 1_{w \geq 1}$, $\beta > 0$ and edge connection probability between $x, y \in \mathscr{P}_\lambda$ is $1 - \exp\{-\eta \frac{W_x W_y}{|x-y|^\alpha}\}$, $\eta, \alpha > 0$, independent of everything else. The eRCM (IeRCM resp.) is obtained by considering two points to be neighbors if there is an edge between them or the edges emanating from them in the RCM (IRCM resp.) intersects. In the Poisson stick model there are sticks centered at each point of $\mathscr{P}_\lambda$, having uniform orientation and length random variable having unbounded support. The points in $\mathscr{P}_\lambda$ are said to be neighbors if the corresponding sticks intersects.

The model is said to percolate if there is an infinite connected component. We derive condition on the connection function so that the model exhibits a non-trivial phase transition, that is, there exists a $\lambda_c \in (0,\infty)$ such that for $\lambda > \lambda_c$ the model percolates and for $\lambda < \lambda_c$ the model does not percolate. We derive a Russo-Seymour-Welsh (RSW) lemma using which we prove the condition on the connection function under which no percolation occurs at criticality.

The RSW lemma gives us information about the probability of having a crossing path from left to right along the longer side of a rectangle $[0, \rho n] \times [0, n]$, for any $\rho \geq 1$, when we have information about the probability of having a left to right crossing path in a square $[0, n] \times [0, n]$. The difficulty in proving the RSW lemma is that the edges can be arbitrarily long. The result on longest edge length enables us to overcome the difficulty.

Filip Seitl

Exploration of Gibbs-Laguerre tessellations for 3D stochastic modeling

Joint with L. Petrich, C. E. Krill III, V. Schmidt, J. Staněk and V. Beneš

Random tessellations generated by Gibbs point processes are investigated. The motivation comes from the materials research where the 3D grain structure of polycrystalline metals is investigated. The Gibbs-Voronoi tessellations in 2D were examined in [1]. We will deal with a twofold extension of this concept, namely to Gibbs-Laguerre tessellation in 3D. The existence of this model can be treated by methods in [2]. The choice of the energy function of the underlying Gibbs point process reflects desired geometrical characteristics of grains. Simulations using MCMC are tractable. The model is applied to statistical reconstruction, [3], based on a real data specimen of a polycrystal.

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Jakub Staněk

Similarity of realisations of random sets via their morphological skeletons

Joint with Johan Debayle, Vesna Gotovac, Kateřina Helisová and Markéta Zikmundová

The poster concerns a method of assessing similarity of realisations of random sets based on construction of their morphological skeletons and consequent covering of the realisations by unions of discs with centres on the skeletons. Since the realisations are considered to be binary images, the skeletons together with the corresponding discs can be viewed as marked point processes with specific properties. Different functions for comparing such marked point processes are shown. The described procedure is illustrated on a simulation study with the aim to distinguish between realisations coming from different models.

Helene Svane

Reconstruction of r-regular objects from images with or without noise

Joint with Andrew du Plessis and Aasa Feragen

We study what information can be retrieved from digital images of planar objects. The images that we consider are made from an object by placing the object in a grid an then colouring pixels completely covered by the object black, pixels partly covered by the object grey, and the remaining pixels white. The goal is to use digital images obtained this way to reconstruct the original object, or at least to construct an object similar to it. To do so, we need to assume some regularity on the objects that we study. The regularity constraint that we will use is called r-regularity.

A related problem is to reconstruct r-regular objects from their noisy images. We propose a method for exploiting the properties of non-noisy images of r-regular sets to reconstruct the original r-regular object from its noisy image in this more general setting.

Riccardo Turin

Limit theorem for floating-like bodies.

Joint with Ilya Molchanov

With each convex body K in Euclidean space, it is possible to associate its floating body K_δ obtained by cutting away all caps of given volume $\delta > 0$. A variant of this construction derives subsets of K from a measure on K. Namely, as in [1], for any Borel measure μ on $\mathbb{R}^d$ it is possible to consider the convex set of all points of the form

$$\int_{\mathbb{R}^d} y f(y)\mu(dy)$$

where $f : \mathbb{R}^d \to [0,1]$ is μ-measurable, $\int_{\mathbb{R}^d} f(y)\mu(dy) = 1$ and such that the above integral is defined. If μ is the uniform measure on K of total mass δ^{-1}, one obtains Ulam's floating body that lies between K_δ and K (see [2]).

In this work we suggest a fairly general scheme producing floating-like bodies with the help of sublinear expectations of random vectors. Furthermore, we consider empirical variants of floating-like bodies generated by i.i.d. samples. We prove the strong law of large numbers for the empirical floating-like bodies and the law of the iterated logarithm for the case of empirical convex floating bodies.

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Irina Volchenkova

A new definition of random sets

Joint with Vesna Gotovac, Kateřina Helisová and Lev B. Klebanov

Mathematical theory of random sets is very popular nowadays. It has found its application in many fields of science such as physics, biology, medicine etc.

In this poster, we define random sets that are random elements taking values in Boolean algebra (B.A.) with a finite (or just probabilistic) measure on it. Of course, such sets may be considered as subsets from a realization of B.A. as a clopen subsets algebra of the totally disconnected compact space.

Our approach starts from a normed B.A., i.e. a complete B.A. endowed with a positive finite measure. Without loss of generality we can suppose that this is probabilistic measure, or, shortly, probability. It allows one to introduce a distance on B.A., which will be used to define class of Borel subsets of B.A.

First, we introduce the notion of the Boolean algebra of the sets. Then, there follows brief introduction to the theory of positive and negative definite kernels and $\mathcal{N}$—distances. Further on we describe the embedding of normed B.A. into a Hilbert space of the functions using positive and negative definite kernels. The new notion of a random set is introduced together with the characteristic of its distributions. A $\mathcal{N}$—distance on the space of this characteristic is constructed here. Also, some new operations on the characteristics are defined. After that, we provide some results concerning limit theorems for the discrete random sets obtained via pointwise convergence of the characteristics of their distributions. Finally, there are given some possibilities for applying newly developed theory to statistical testing.