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Abstract

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Boundary effect on the nodal length for Arithmetic Random Waves, and spectral semi-correlations

Joint with Oleksiy Klurman and Igor Wigman

We test M. Berry's ansatz on nodal deficiency in presence of boundary. The square billiard is studied, where the high spectral degeneracies allow for the introduction of a Gaussian ensemble of random Laplace eigenfunctions ("boundary-adapted arithmetic random waves"). As a result of a precise asymptotic analysis, two terms in the asymptotic expansion of the expected nodal length are derived, in the high energy limit along a generic sequence of energy levels. It is found that the precise nodal deficiency or surplus of the nodal length depends on arithmetic properties of the energy levels, in an explicit way.

To obtain the said results we apply the Kac-Rice method for computing the expected nodal length of a Gaussian random field. Such an application uncovers major obstacles, e.g. the occurrence of "bad" subdomains, that, one hopes, contribute insignificantly to the nodal length. Fortunately, we were able to reduce this contribution to a number theoretic question of counting the "spectral semi-correlations", a concept joining the likes of "spectral correlations" and "spectral quasi-correlations" in having impact on the nodal length for arithmetic dynamical systems.

This work rests on several breakthrough techniques of J. Bourgain, whose interest in the subject helped shaping it to high extent, and whose fundamental work on spectral correlations, joint with E. Bombieri, has had a crucial impact on the field.