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Abstract

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The fractal cylinder model

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We consider a statistically semi-scale invariant collection of bi-infinite cylinders in $\mathbb{R}^d$, chosen according to a Poisson line process of intensity λ. The complement of the union of these cylinders is a random fractal which we denote by $\mathcal{V}$. This fractal exhibits long-range dependence, complicating its analysis.

Nevertheless, we show that this random fractal undergoes two different phase transitions. First and foremost we determine the critical value of λ for which $\mathcal{V}$ is non-empty.

We additionally show that for $d \geq 4$ this random fractal exhibits a connectivity phase transition in the sense that the random fractal is not totally disconnected for lambda small enough but positive.

For $d = 3$ we obtain a partial result showing that the fractal restricted to a fixed plane is always totally disconnected.

An important tool in understanding the connectivity phase transition is the study of a continuum percolation model which we call the fractal random ellipsoid model. This model is obtained as the intersection between the semi-scale invariant Poisson cylinder model and a k-dimensional linear subspace of $\mathbb{R}^d$. Moreover, this model can be understood as a Poisson point process in its own right with intensity measure $\ell_k \times \mu_E$ where ℓ_k denotes the Lebesgue measure on $\mathbb{R}^k$ and μ_E is the shape measure describing the random ellipsoid.