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Abstract

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Some older and newer results on Brillinger-mixing point processes

We consider a stationary infinite-order point process $\Psi \sim P$ on $\mathbb{R}^d$ satisfying the additional assumption that, for each $k \geq 2$, the reduced kth-order factorial cumulant measure $\gamma_{red}^{(k)}(\cdot)$ has finite total variation $||\gamma_{red}^{(k)}||_{TV}$ on $(\mathbb{R}^d)^{k-1}$. This property of Ψ, which is attributed to D. R. Brillinger, expresses weak mutual correlations between the numbers of atoms taking the counting measure Ψ in distant sets. This condition allows to prove asymptotic normality of shot noise processes, higher-order moment measure estimators (e.g. the empirical K—function), empirical product densities etc. Furthermore, CLTs for hyperplane processes driven by a one-dimensional B-mixing pp can be established. $\Psi \sim P$ is called *strongly B-mixing* if $||\gamma_{red}^{(k)}||_{TV} \leq a^k\, k!$ for some $a > 0$ and any $k \geq 2$. We show that a *B-mixing* pp $\Psi \sim P$ is *mixing* iff P is uniquely determined by its one-dimensional moment sequences. If there is no uniqueness then $\Psi \sim P$ need not be *ergodic* in general, see [1]. On the other hand, if the pp $\Psi \sim P$ is strongly B-mixing then its tail-σ-algebra of Ψ is trivial. Finally, we show that a Log-Gaussian Cox process is B-mixing if the covariance function $c(\cdot)$ of the underlying stationary Gaussian field is absolutely integrable and conditions are formulated implying that a renewal pp is strongly B-mixing.

References

[1] L Heinrich (2018) Brillinger-mixing point processes need not to be ergodic. *Stat and Prob Letters* **138**, 31–55.