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Abstract

Jesper Møller

The structure of stationary time series and point processes when constructing singular distribution functions

Joint with Horia Cornean, Ira W. Herbst, Benjamin Støttrup and Kasper S. Sørensen

A function $F:\mathbb{R}\to\mathbb{R}$ is singular if it is non-constant and $F'(x)=0$ for Lebesgue almost all $x\in\mathbb{R}$ (sometimes further conditions are imposed). We construct rich classes of singular cumulative distribution functions (CDF) F for random variables X on the unit interval $[0,1]$. Our basic idea is to construct such functions by considering a q-adic expansion of X, where $q\in\mathbb{N}$, and where the coefficients in the expansion form a time series as follows. Let $X_1,X_2,\ldots$ be a time series with values in $\{0,1,\ldots,q-1\}$. Then define $X:=\sum_{n=1}^{\infty}X_nq^{-n}$.

In particular, we completely characterize the CDF when $\{X_n\}_1^\infty$ is stationary; or equivalently when the point process $\sum_1^\infty\delta_{X_n}(\cdot)$ is stationary (here, δ_t is the Dirac measure at t). In fact F becomes a mixture of three CDFs F_1,F_2,F_3 on $[0,1]$, where F_1 is the uniform CDF on $[0,1]$; F_2 is singular discrete and is a mixture of countable many CDFs, each of them being uniform on a finite set of so-called purely repeating q-adic numbers which are members of a cycle (we clarify how this corresponds to a stationary cyclic Markov chain of order equal to the length of the cycle); and F_3 is singular continuous.

Two simple models are well-known: Take $\{X_n\}_{n\geq 1}$ to be independent identically distributed. In the dyadic case $q=2$, if 0 and 1 equally likely, then $\mathrm{d}F$ is just Lebesgue measure on $[0,1]$. In the triadic case $q=3$, if 0 and 2 are equally likely and $\mathrm{P}(X_1=1)=0$, then F is the well known Cantor function.

These models and several others are discussed more fully in the talk. Moreover, we demonstrate in several examples that continuity of F is a natural property when considering specific models for stationary time series and point processes.