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Abstract

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Point process generative model with wavelet phase harmonics

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This work aims at presenting a method for tackling the following problem: given the finite realization ϕ of a planar point process $\Phi \in \mathbb{M}(\mathbb{R}^2)$ with unknown distribution, we want to generate (approximations of) realizations of Φ. Based on the assumptions that the sample ϕ is large enough and that Φ is ergodic, our approach consists in building a family of descriptors $\{S_k(\phi),\ k \in \Lambda\}$, such that $S_k(\phi) \simeq \mathbb{E}(S_k(\Phi))$. If the descriptors are informative enough, they characterize the distribution, and any $\mu \in \mathbb{M}(\mathbb{R}^2)$ satisfying $S_k(\mu) \simeq \mathbb{E}(S_k(\Phi))$ is a good approximation of a realization of Φ. We then generate an approximate sample of Φ by first generating a sample $\nu = \sum_i \delta_{x_i}$ from an homogeneous Poisson point process, and use a gradient descent algorithm to find the positions $\{x_i^*\}$ that minimize $\sum_k |S_k(\phi) - S_k(\sum_i \delta_{x_i^*})|^2$. The descriptors we use in this work are based on wavelet transform, and are called wavelet phase harmonics [1]. They estimate the correlation of wavelet transforms across scales, with frequency transpositions. These descriptors are related to the correlations of feature maps of non-linear rectifier layers in deep neural networks. We study their ability to capture the distribution of highly structured (Poisson cluster, Cox) point processes, as well as the pertinence of the gradient descent method in generating approximations of point processes of a given distribution, by computing several statistics for the generated samples. The moduli of the wavelet transforms, called scattering moments, have already been used in [2] to statistical learning of geometric marks of point processes.

References

[1] S. Mallat, S. Zhang and G. Rochette (2018) Phase Harmonics and Correlation Invariants in Convolutional Neural Networks. *arXiv:1810.12136*

[2] A. Brochard, B. Błaszczyszyn, S. Mallat, S. Zhang (2018) Statistical learning of geometric characteristics of wireless networks. *arXiv:1812.08265* To appear in *Proc. of IEEE INFOCOM 2019*