On spatio-temporal Quermass-interaction process

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joint work with Markéta Zikmundová [2] *and Viktor Beneš* [2]

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Abstract

Consider a random set observed in a bounded window $W \subset \mathbf{R}^2$ in discrete times $k = 0, 1, ..., T$. The set is given by a union of interacting discs and it is developed in time so that the discs appear and disappear (but do not grow). In each time k, the set is described by the Quermass-interaction process, i.e. the probability density of any finite configuration $\mathbf{x} = (x_1, ..., x_n)$ of the discs $x_1, ..., x_n$ with respect to the probability measure of a stationary random-disc Boolean model is given by

$$f_{\theta^{(k)}}(\mathbf{x}) = \frac{\exp\{\theta_1^{(k)} A(U_{\mathbf{x}}) + \theta_2^{(k)} L(U_{\mathbf{x}}) + \theta_3^{(k)} \chi(U_{\mathbf{x}})\}}{c_{\theta^{(k)}}}, \tag{1}$$

where $A(U_{\mathbf{x}})$ denotes the area, $L(U_{\mathbf{x}})$ the perimeter and $\chi(U_{\mathbf{x}})$ the Euler-Poincare characteristic of the union $U_{\mathbf{x}}$ composed of the discs from the configuration $\mathbf{x}$. Further, for each time k, $\theta^{(k)} = (\theta_1^{(k)}, \theta_2^{(k)}, \theta_3^{(k)})$ is a vector of parameters and $c_{\theta^{(k)}}$ is a normalizing constant.

The temporal evolution of the random set is given by the evolution of the parameters according to the relation

$$\theta^{(k)} = \theta^{(k-1)} + \eta^{(k)},\ k = 1, 2 \ldots, T, \tag{2}$$

where $\theta^{(0)}$ fixed is given and $\eta^{(k)}$ are iid random vectors with Gaussian distribution $\mathcal{N}(a, \sigma^2 I)$, where $a \in \mathbf{R}^3, \sigma^2 > 0$ and I is the unit matrix.

The temporal dependence in the random set is defined within its simulation algorithm. We start the simulation so that we choose a fixed $\theta^{(0)}$

and according to (2), we simulate parameter vectors $\theta^{(k)}, k = 1, 2, \ldots, T$. Further, using classical birth-death Metropolis-Hastings algorithm MCMC (see [1]), we simulate a realization $\mathbf{x}_0$ from the density (1) with $\theta^{(0)}$. Then we simulate realizations $\mathbf{x}_k$, $k = 1, 2 \ldots, T$ from the density (1) with $\theta^{(k)}$ and the birth-death Metropolis-Hastings algorithm is used again, but with a special way of adding a disc: since the realizations are aimed to be dependent, the choice of a newly added disc in the algorithm depends on the previously simulated configuration $\mathbf{x}_{k-1}$ so that the proposal distribution of the newly added disc $Prop_k$ at time k is a mixture

$$Prop_k = (1 - \beta) \cdot Prop^{(RP)} + \beta \cdot Prop_{k-1}^{(emp)}, \quad \beta \in (0, 1),$$

where $Prop^{(RP)}$ is a distribution of the reference process, $Prop_{k-1}^{(emp)}$ is the empirical distribution obtained from the configuration $\mathbf{x}_{k-1}$ and β is a chosen constant describing power of time dependence. It means that $(\beta \times 100)\%$ of the added discs are taken from the previous configuration and the remaining discs are simulated randomly, so the dependence is stronger when β is bigger.

In this contribution, different methods for estimating the parameters $\theta^{(k)} = (\theta_1^{(k)}, \theta_2^{(k)}, \theta_3^{(k)})$, $a = (a_1, a_2, a_3)$ and σ^2 will be described. More precisely, combination of MCMC maximum likelihood method described in [2] with regression methods and particle filter studied in [3] and [4] will be shown.

References

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