The cylindrical K-function for anisotropy analysis of two- or three-dimensional point patterns

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Abstract: Detecting anisotropy caused by columnar arrangement of points in two- or three-dimensional point patterns can be summarized by a directional counterpart of Ripley's K-function (based on a spherical test set) called the cylindrical K-function (which is based on a cylindrical test set). Roughly speaking, the cylindrical K-function $K_u(r,t)$ is proportional to the mean number of further points within a cylinder centered at a typical point, when the cylinder has direction u, height $t > 0$, and the base radius $r > 0$. Since it is defined for any direction u in 2D or 3D space, if there is a columnar arrangement, the behaviour of the function is expected to be changed when the direction u is changed, and its value to be larger than the expected value under CSR when u is in accordance to the direction of columns. There is a close relationship between the cylindrical K-function and the space-time (i.e. 2D-space$\times$1D-time) K-function introduced in Diggle *et al.* (1995).
We apply this function to investigate the minicolumn hypothesis in neouroscience using 3D data sets collected by the Bioimaging group at Aarhus University. This project is associated to the Center for Stochastic Geometry and Advanced Bioimaging (CSGB).