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Quasimetric Embeddings and their algorithmic applications

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We study generalizations of classical metric embedding results to the case of *quasimetric* spaces; that is, spaces that do not necessarily satisfy symmetry. Quasimetric spaces arise naturally from the shortest-path distances on directed graphs. Perhaps surprisingly, very little is known about low-distortion embeddings for quasimetric spaces.

Random embeddings into *ultrametric* spaces are arguably one of the most successful geometric tools in the context of algorithm design. We extend this to the quasimetric case as follows. We show that any n-point quasimetric space supported on a graph of treewidth t admits a random embedding into *quasiultrametric* spaces with distortion $O(t \log^2 n)$, where quasiultrametrics are a natural generalization of ultrametrics. This result allows us to obtain $t \log^{O(1)} n$-approximation algorithms for the Directed Non-Bipartite Sparsest-Cut and the Directed Multicut problems on n-vertex graphs of treewidth t, with running time polynomial in both n and t.

The above results are obtained by considering a generalization of random partitions to the quasimetric case, which we refer to as *random quasipartitions*. Using this definition and a construction of [Chuzhoy and Khanna 2009] we derive a polynomial lower bound on the distortion of random embeddings of general quasimetric spaces into quasiultrametric spaces. Finally, we establish a lower bound for embedding the shortest-path quasimetric of a graph G into graphs that exclude G as a minor. This lower bound is used to show that several embedding results from the metric case do not have natural analogues in the quasimetric setting.